2021

High school Mathematical Contest in Modeling (HiMCM)
Summary Sheet
Team Control Number: 11900
Problem Chosen: A

Summary

The usage of solar power has, in recent years, been adopted by many countries to reduce CO_2 emission. However, compared to traditional methods of producing power, solar power is not capable of producing power consistently as it is heavily dependent on the sun. A method to store solar energy is therefore necessary to store solar energy when available, and to release it when power is needed. The most common way to store electricity is to use a battery. However, batteries come in different costs and qualities, and a method is necessary to determine how good a battery is, and what battery is the best.

Therefore, we use the Analytic Hierarchy Process (AHP). This method is suitable to the problem as it gives weight from the comparative importance of the factors, which may vary due to different geological issues. In generalizing the model, we choose several locations on Earth that have distinct characteristics. These characteristics, as they alternate the comparative importance of factors over each other, affect the ultimate choice of the best battery.

In short, we believe that the Discover AES 7.4kWh battery is the best battery due to its excellent instantaneous power ratings and round-trip efficiency. Its continuous power rating and capacity is also good while having medium cost. This would be of use both in warm countries, where the instantaneous power rating is necessary to power air conditioners; and in colder countries, where capacity and efficiency are important in making the most out of the minimal sunshine. The choice of this battery, therefore, would be the best choice. Another alternative to this choice is the Tesla Powerwall + battery. It also has good qualities and excellent capacity. However, due to its higher cost and bulkiness, it isn't as good as the Discover battery. The rest of the batteries were not a good choice, either because of poor quality or high cost.

The demand for electricity generally becomes greater with the advancement in technology and living conditions. In 2020, the average annual electricity consumption for a U.S. residential utility customer was 10,715 kilowatt hours (kWh). A typical U.S. home consists of lightings, a dryer, a stove, a water heater, a dishwasher, a disposal, and a central AC.

Another alternative to using traditional batteries is cement-based battery. One of the most updated research on sustainable energy and solar energy batteries has developed and improved on the cement-based batteries by increasing energy density. It is suggested that with the installation of this technology, the ubiquitous application of concrete would provide extra capacity for home use energy storage. Yet, after close examination and calculation of the given data, as well as the comparison with other choices in the market, we hold an overall negative altitude for its application in the current stage. Its disadvantage includes low storage capacity and high costs. Furthermore, information including potential dangers, performance under extreme conditions, and long-term durability was not provided for more rounded decision making. Nevertheless, we are optimistic for the future developments of this technology.

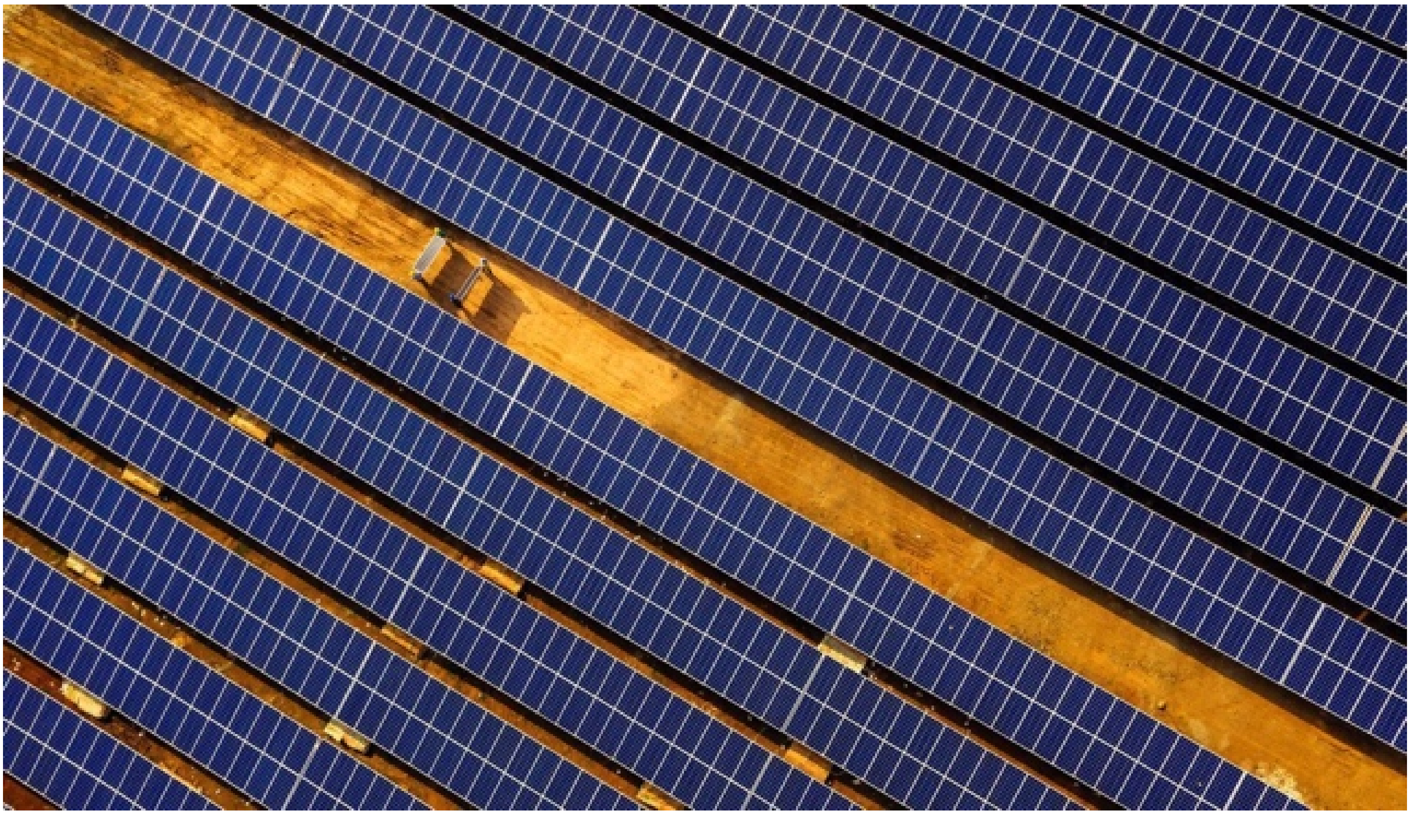


Photo by Tuấn Nguyễn, submitted to National Geographics

Choices for Home-Use Batteries Explained and Designed for You.

Scholars and political leaders from over 200 countries finally reached the consensus on preventing global warming and environmental protection this Saturday on the UN COP26 summit held in Glasgow, UK. Scientists and governments around the world have been seeking alternatives for conventional fossil fuels, experimenting with renewable sources including solar and tidal energy. Along with the trend of finding greener solutions, demand for solar storage batteries have skyrocketed. Yet, baffled by the numerous measures with both pros and cons, ordinary consumers may face difficulties in making their purchasing decisions. But now, with the recent studies and models developed by our research team, you may locate the best suitable one among seas of battery choices to meet their personalized requirements.

Our team uses mathematical approach to process the data of different determining factors. Our modified AHP model helps you identify your personal needs, and then compare the criteria by ranking a priority order

based on algorisms. We would specifically focus on external factors including consumers' household electric devices and the climate characteristics of the houses' locations. In this way, with sufficient information and properties of available battery choices, you may find their best choices by simply inserting their vague preferences.

Recently, researchers have incorporated solar energy batteries into the most common construction material—cement. In terms of sustainable growth and maximizing efficiency, this new technological breakthrough provides us with advanced concepts of future designs. Yet, as a suggestion to all consumers based on our strict and thorough evaluation, the cement-based battery has many downsides as well, including insufficient storage capacity (for smaller houses), unsolved durability measures, and potential safety concerns. Nevertheless, on the bright side we are facing a future of infinitive possibilities, and this new technology may trigger waves of future improvements and commercialization.

For detailed description and analysis of our model and cement-based batteries, please follow us into the sections upcoming.

Contents

1. Introduction

1.1 Background

Humankind, propelling the rapid growth of industry, has committed relentless exploitation of Earth's environment since the last century, from the emission of greenhouse gases that drives the temperature beyond control, to the construction of mass infrastructures that disturb the balance of untouched yet fragile ecosystems. In the 2021 COP26 summit held in Glasgow, U.K., global political leaders and scientists have reached consensus of the severity and urgent demand for scientific innovations to address this global crisis.[1] Cleaner and greener solution to the energy crisis has long been coveted by officials, including the ubiquitous and highly efficient solar energy.

Yet, residents and scientists encounter many natural barriers when installing and improving solar energy technologies. In United States, the unpredictable natural calamities and outdated power grid systems in some regions stresses the continuity and capacity of energy storage by using batteries. When the ruthless snowstorm stuck Texas in February 2021, millions of residents were left helpless in permanent darkness and frigidity for weeks due to the breakdown of electricity plants and power grids. If with sufficient power stored in batteries with high efficiency of energy conversion, the size of the tragedy may shrink prominently. Yet, factors including capacity, continuous and instantaneous power rating, round-trip efficiency, and of course, cost, trouble consumers in terms of making choices between different available batteries.

1.2 Question Restatement

The purpose of our project is to provide cost-efficient purchasing plan of home-use off-grid batteries (choosing from the batteries provided in the question) by constructing a mathematic model, taking both internal factors (e.g., storing capacity, energy discharge efficiency) and external factors (e.g., socio-economic levels, number of sunny days in different seasons).

Question 1a: We need to consider factors including house population, number and power of electricity products, as well as the time for dense electricity use. These questions lead us to the construction of model by offering samples of factors to consider.

Question 1b and 1c: By using the factors mentioned in 1a, we are required to construct a mathematical model to offer quantitative analysis of purchasing decisions. We would use AHP(Analytical Hierarchy Process) model to rank the priority of different factors and consider the use of EWM (Entropy Weight Method) to reduce effects of subjectivity, then give overall ratings for all choices offered. Detailed explanation of the model and result analysis is given.

Question 2: Beside the factors discussed above, there are much more determinants for making battery purchasing decisions. Thus, by modifying the model

---

[1] Ellyat, Holly. 2021 Nov 9th. *COP26 climate summit switches to science and innovation*. CNBC. https://www.cnbc.com/2021/11/09/cop26-updates-as-climate-summit-continues-in-glasgow.html?&qsearchterm=COP26

constructed for 1b and 1c and incorporating factors regarding personalized demand of consumers around the world, we made a general suggestion among available choices. Specific reasoning is shown.

Question 3a and 3b: The newest update of cement-based batteries provides consumers with additional electricity storage. Yet, this experiment is in its initial stages, which means crucial determinants of its full commercialization is not available. We would fully analyze the given information and comment on its application.

1.3 Our work

We have incorporated analytical comparison of factors into the AHP (Analytic Hierarchy Process) modeling method to receive the priority rankings in a presumed situation. Then, the effects of external factors were incorporated into the model to yield a general solution. Finally, the best-suitable battery among the listed choices was suggested based on the calculations.

In the initial step of information gathering, we investigated the basic principles behind batteries and the various factors affecting their performance. Meanwhile, we obtained geographical and meteorological data from 10 different locations around the world possessing distinct climate and socio-economic levels, in order to fully analyze the preference of consumers.

After that, we can compare the importance based on qualitative measures that feature consumers' requirements and demands. Then, the results of the comparisons were given specific index indicating the relative importance, which were later processed using the AHP model. Thus, we may rank the priorities for different factors, and based on that, the most cost-efficient choice of battery is suggested to consumers with distinct backgrounds. The rankings are then tested using EWM model to increase accuracy, yet eventually we chose not to use apply this model. Furthermore, based on the recent research of concrete electrolytic cells, we proceed to discuss the possibilities and downsides of applying this new technology.

2. Assumptions and justifications

Assumption 1: The off-the-grid system supplies energy for a family of three members.

Justification 1: According to US Energy Information Administration (EIA)[2], the 118.2 million families in US have 2.48 people in each on average. The majority (42.7 million) of the families consist of 2. In order to cope with the need of families in a wider range, the family being served by off-the-grid energy storage system is assumed to have three members.

Assumption 2: The total power consumption of the family per month is 900kwh[3].

Justification 2: The research taken in 2020 conducted by EIA suggests that the annual electricity consumption for a regular US family was 10715 kilowatt hours, which is about 893 kWh per month. The value varies significantly depending on

---

[2] https://www.eia.gov/consumption/residential/data/2015/hc/php/hc9.4.php [Accessed on Nov.12, 2021]

[3] https://www.eia.gov/tools/faqs/faq.php?id=97&t=3 [Accessed on Nov.12, 2021]

geographical factors. Hawaii had a monthly consumption of 537 kWh, whereas families in Louisiana used up 1201 kWh per month in average. In order to accommodate to various housing conditions, the off-the-grid energy storage system should be capable of generating at least 900 kWh per month.

Assumption 3: The off-the-grid energy storage system must be capable of performing under the load of 40 kW

Justification 3: Based on research on common American families, we have built a chart with major appliances.

Assumption 4: For the Deka Solar 8GCC2 6V 198 battery and the Trojan L-16 -SPRE 6V 415battery, the instantaneous power rating is equal to the continuous power rating.

Justification 4: The instantaneous power ratings of the batteries are not available. Therefore, they should be equal to the continuous power rating, which is the minimum.

Assumption 5: The time for which the instantaneous power rating is available is not important.

Justification 5: The power surge when opening a device typically doesn't last long.

1. General Electrical Load Requirements			
Appliance	Quantity	Power rating	load
Lighting	1600ft^2	3W/ft^2	4.8kW
2. Motor Loads			
Dryer	1	5kW	5kW
Stove	1	8kW	8kW
Water heater	1	4.5kW	4.5kW
Dishwasher	1	1.2kW	1.2kW
Disposal	1	0.6kW	0.6kW
3. Heating and Air Conditioning			
Central AC	1	8.5kW	8.5kW
Calculated Total Load:			40kW

An electrical load calculator determines the size of the electrical service based. upon the electrical equipment installed. Under the assistance of Electrical Load Calculator[4], required appliance circuits and other parameters are taken under considerations to simulate an actual maximum load. Thus, the system is expected to carry out 40kW load with 100% power factor.

3.Basic Model for Considered Variables

---

[4] https://ask-the-electrician.com/residential-electrical-load-calculation.html#beginAdv [Accessed on Nov.12, 2021]

3.1 Basic info

In order to solve the problems, we decide to use AHP, analytical hierarchy process, in order to solve a complex problem. AHP is based on mathematics and psychology. It could calculate the accurate weights of each criterion then determine the possible alternatives for the problem. In our task, we need to analyze which battery type to use to meet the requires for off-the-Grid electricity circuit. Off-the-Gid circuit is always used in emergency situations, so the criterion to choose the final type(s) of battery is rather strict. The following shows how we use AHP to approach the choice

3.2 Case application

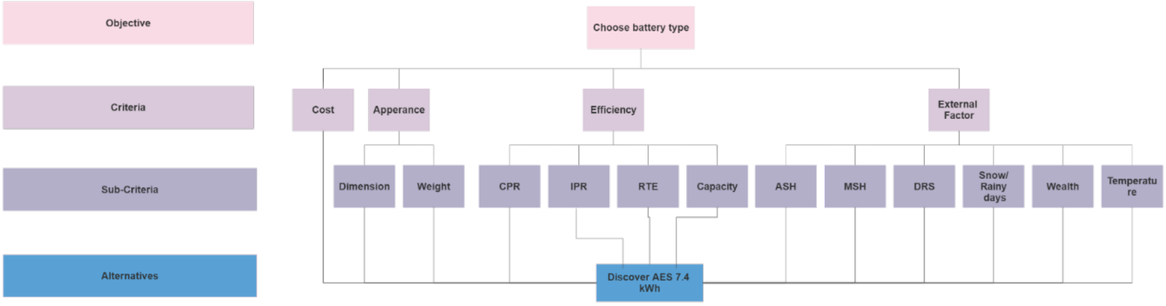


Figure 1: The hierarchy structure of variables

AHP has three hierarchies, which are the “Objective”, “Criteria”, and “Alternatives”. In our investigation, our objective would be choosing the battery type, and we would reach our goal under four main criteria: Cost, appearance, efficiency, and external factor. And these criteria have several sub-criteria. And with these sub-criteria, we could eventually determine the certain type(s) of battery that best fit the requirements of the off-the-Grid circuit. But here’s a question. How do we determine the weights of each criterion with so many different categories? Then we would have to use the core thought of AHP.

3.3 Criteria determination

The reasons for the decision of the weights we made are explained in the section below.

- **Internal determinants**

3.3.1 Cost

A very important determinant of a good battery is its cost. Obviously, it would be good if a battery is cheap while still having good qualities. Though other factors are also important in determining how good a battery, cost, to many consumers, are of their top concern.

3.3.2 Weight

How heavy a battery is will result in how much it could pile, which would be crucial if the space for storage is cramped and limited. If a battery has too much weight exerted on it, it will perform under maximum capacity, or even break. Therefore, a lighter battery could bring consumers greater electricity storage or more solutions.

3.3.3 Bottom surface area

The bottom surface area of a battery will determine how much batteries can be put into the land without being piled together. A bigger the battery will result in fewer batteries in a piece of fixed area, and thus greater cost-efficiency.

3.3.4 Height

The height of a battery will determine how much of it can be piled together. Since the total height of the batteries cannot exceed a certain height, a battery of smaller height will mean that more batteries can be piled together.

3.3.5 Continuous battery ratings

Continuous battery rating is the measure of rate of battery discharge relative to its largest capacity. In other words, it determines how many electric devices can be powered at the same time, and for how long. A higher continuous battery rating will enable more electric devices to be active at the same time and will mean a better battery. It is measured in unit of C, whereas $1C$ means that the discharge current would discharge all its electricity in 1 hour.

3.3.6 Instantaneous battery ratings

When switching on certain devices, for example an air conditioner, the sudden battery output may, for a short length of time, be very large. Therefore, a battery with a higher instantaneous battery rating will be able to accommodate a higher sudden output and will be less susceptible to fusing accidents.

3.3.7 Efficiency

Solar power is not a very efficient way of producing electricity, and efficiency is valuable in making the most out of the solar power. A battery with higher round trip efficiency will waste less energy, indicating that more electric energy would be successfully converted to other types of energy, and it will be a better choice.

3.3.8 Capacity

By installing solar panels, we may collect solar energy and transfer them into electricity. However, when the sun is not available, for example during nights or rainy days, energy use depends on the stored capacities. A battery with more storage capacity provides sufficient energy for emergency use, especially in rainstorms or natural disasters to support basic life.

- **External Determinants**

(For question 2)

3.3.9 Average sunshine hours

Annual average sunshine hours per day determines the average amount of electricity obtained from collecting solar energy, which were then stored in the battery for uses at night or other occasions. Thus, for regions with shorter annual average sunshine hours, higher round-trip efficiency and storage capacity of batteries are required to cope with potential shortages. One determinant of average sunshine hours would be the locations' latitude; those located closer to the equator receives more direct solar radiation.

3.3.10 Minimum sunshine hours

While the annual average sunshine hours per day provides us with the overall picture of the access to solar powers, daily and seasonal sunshine hours reflects the specific requirements that stand out. Countries in the northern hemisphere, have nights longer than days during winter due to the Earth's position relative to the sun. The shortest sunshine hours occur on the winter solstice day. Countries within the Arctic circle experience nighttime of over 24hours in winter known as the polar night effect. Batteries with high round-trip efficiencies, high capacities are required to spend the long nights.

3.3.11 Distinct rain seasons

Same as the sunshine hours, rain seasons affect the collection of solar energy and electricity. Resulted from ocean currents and monsoons, rain seasons may last for up to 4 months of heavy, relentless precipitation and few sunny days. This phenomenon is distinct in regions close to equator, namely South and South-Eastern Asia. For example, during the Indian monsoon stretching from June to August, rainfall over the Deccan varies from 400mm to 500mm[5], which is about 200% of which in the dry seasons. Without sufficient electricity support, residents may face risks of desolation and helpless flooding strikes. Thus, sufficient storage and high continuous power rating is vital to sustain livings.

3.3.12 Average number of snow/rainy days

On snowy and rainy days, families living off-the-grid could only depend on battery for electricity. Regions closer to the poles experience longer winters, and thus face striking demand for heating with limited solar energy access. Cloudy and rainy days abound in regions belong to the Temperate Marine Climate zone, which stretches along the western coast of North America and Europe; residents in those regions would like to have batteries with high capacity and continuous power ratings.

3.3.13 Wealth level

To measure the wealth level and economic health of the regions we selected, their respective annual GDP (Gross Domestic Product) are examined. Being the sum of citizens' consumption, investment, government spending, and net export, GDP best reflects the growth rate and life quality of a certain region. This index is crucial for our assumptions and estimations since family size and demand for electricity are closely related with it. For example, families in more economically developed regions tend to own more electric devices and thus has greater demand for battery capacity and discharge rates. While taken for granted as necessities in developed regions, batteries may be recognized as luxury goods in depressed regions, which means price would be weighted significantly while making purchasing decisions.

3.3.14 Maximum and minimum temperature

The maximum and minimum temperature determines the families' demand for air-conditioning devices during summers and winters. Continuous cooling or heating is crucial to sustain life and well-beings. As those machineries require large power to function over relative long periods of time, the importance of instantaneous and continuous power rating is stressed. Furthermore, large capacity of batteries is also coveted.

---

[5] Krishnamurti, T. (2015, October 12). *Indian monsoon*. Encyclopedia Britannica. https://www.britannica.com/science/Indian-monsoon

4. Determination of the weights for factors

For development of qualitative analysis using AHP, each factor was assigned an index to indicate relative importance.

Take the first eight internal determinants as an example. In order to build a matrix, we first grade the factors on a scale from one to ten. Arithmetic mean of the four values from four members are taken to give out a score for each of the factor.

Based on preliminary research revolving the factors, an overall concept of a qualified battery was established. For instance, a measure of 8 was assigned to cost, for it serves as a major determinant in selection, especially under conditions having great demand for batteries like an off-the-grid system. In contrary, the weight has a relatively insignificant effect on functioning. Therefore, it received a mark of 2.5. Relative importance can be drawn based on the numerical values. The cost factor earns an index of three in comparison with the height on the table showing relative importance of all variables.

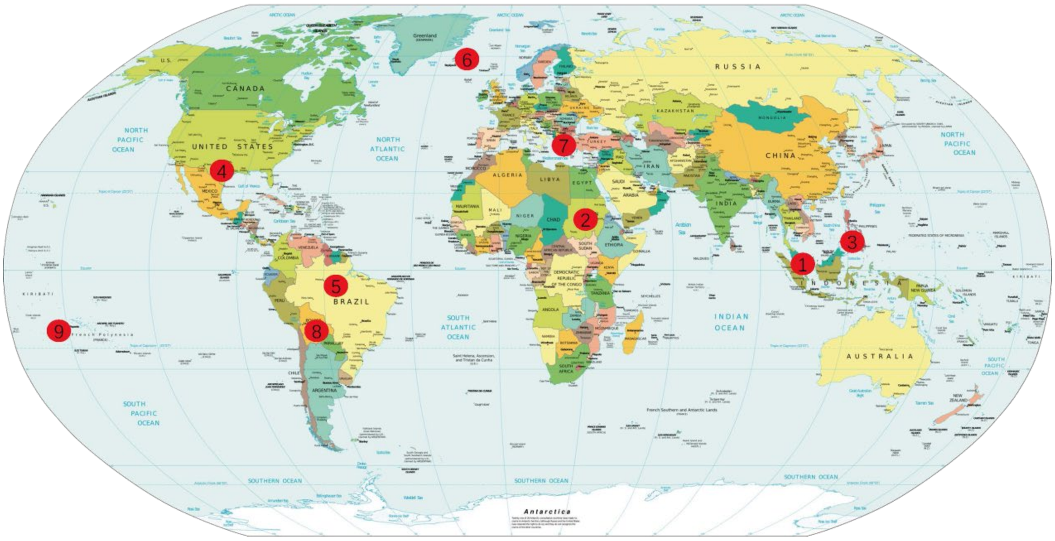
Using the same method, we have concluded a matrix with rows and columns of value for further analysis.

4.1 Effects of geographical factors on weight determination

(For question 2)

In the previous sections we have analyzed battery choice based on moderate climate regions, regardless of extreme weathers or price elasticity for consumers. To answer question 2, we must incorporate various requirements, mostly geographical concerns, into consideration to yield a suggestion for general situations. We have selected nine regions around the world to simulate local markets and demand for different battery properties. To ensure the models best presents the wide spectrum of individual demands, the locations we've selected have great discrepancies in climate, temperature, and wealth levels.

Figure 2. Location selection



1. Singapore 2. Sudan 3. Philippines 4. Texas 5. Brazilian Amazon
6. Iceland 7. Greece 8. Bolivia 9. Society Islands

4.1.1 Cost

To consumers in relatively wealthier regions, the responsiveness of their purchasing desire relative to price changes are not as sensitive as those of consumers in less-developed regions. Thus, we would rank cost at a higher priority for consumers at poorer regions, which is reflected in the GDP per capita shown in Table 1. Thus, for Greece, Texas, Singapore, Iceland and Society Islands, costs are weighted as less significant in their respective AHP models

4.1.2 Continuous and Instantaneous Power Rating (CPR and IPR)

Continuous power rating determines the rate of discharge, or the length of time taken for the battery to fully discharge. Instantaneous power rating measures the power burst in a short period of time. These two criteria are closely related to the number and type of electric devices owned by consumers. Thus, for more developed regions as stated in the last paragraph, higher CPR and IPR are required as they may own more electric devices at home. Generally, we would consider air-conditioning devices to consume more energy than others. As a result, regions with extreme temperatures at summers and winters would also require higher of both these two ratings. All the regions given experience extreme hotness or coldness, and thus would stress the weight of CPR and IPR in the models.

4.1.3 Usable Capacity

Families living in regions with distinct rain seasons or less sunshine require greater battery capacity to support daily use. Thus, for countries located in tropical areas including Brazil, Philippines, Singapore and Society Islands, capacity would be ranked higher in terms of priority. Furthermore, for regions with high latitudes, sunshine hours and solar radiation decrease in winters, affecting the demand for capacity in countries like Iceland.

4.1.4 Size and dimensions

Requirements for battery size and dimensions are affected by accessible space. This would not be a great concern for inland countries, but of a big deal to residents on islands, where resources are often scarce and population being denser. This factor would be ranked as more important in Iceland, Greece, Philippines, Singapore and Society Islands.

Region	GDP per capita of 2020 (in US$)
Brazilian Amazon (Federative Republic of Brazil)	6,796.8
Iceland	59,270.2
Greece	17,676.2
Republic of the Philippines	3,298.8
Republic of Singapore	59,797.8
Texas (United States of America)	63,543.6
Society Islands (French Polynesia)	14,324.1
Republic of Sudan	595.5
South Bolivian Deserts (Plurinational State of Bolivia)	3,143.0

GDP per capital of 2020. Source: The World Bank national accounts data

https://data.worldbank.org/indicator/NY.GDP.PCAP.CD

5. The AHP model

5.1 Structure

First and foremost, it is required that the structure of AHP is clear and known. Thus, in our case, only the objectives, sub-criteria and the alternatives are taken into account. The criteria level is not considered as well since the weights for each upper criterion are hard to determine. The determination of weights of the sub-criteria should be way clearer and easier. Before using AHP, the weights for each sub-criterion on the objective should be discussed previously. We'll first grade each sub-criterion from the importance or effects on the objective, choosing the battery type. * "Cost" here is seen as a sub-criterion.

5.2 Matrix

Then the weights should be shown in a matrix consist of 8 columns and 8 rows, with each row or column represents a sub-criterion(variable). Preference of AHP is based on pairwise comparisons. Each element in the matrix shows a relation of weights comparison.

Here is the scale of criterion we use to create the matrix.

Number	Definition
1	The two factors are of equal importance
3	Factor i is somewhat more important than factor j
5	Factor i is considerably more important than factor j
7	Factor i is significantly more important than factor j
9	Factor i is of the utmost importance compared to factor j
2,4,6,8	A median of the above descriptions
Reciprocals of the above values	A reverse importance (The importance of factor j compared to factor i)
1.1-1.9	Factor i is only slightly more important than factor j

Source: *Decision making for Leaders: The Analytic Hierarchy Process for decisions in a complex world*[6]

For example, C_1, the "Cost" is 4 times important as C_3, the "weight", for example, in our case. It could be indicated that C_3 is 1/4 important as C_1. In the matrix, it is shown by $C_{13} = 4$; $C_{31} = 1/4$. For each element in the matrix, C_{ij} represents the importance of C_i relative to C_j. As a result,

$$[C_{ij}], \text{ where } i, j = 1, 2, ..., n ,$$

$$C_{ij} = 1 \text{ for } i = j ,$$

$$C_{ij} = \frac{1}{C_{ji}}$$

With the base of the scale table, we could establish a matrix for the variables/

---

[6] SAATY T., *Decision making for Leaders: The Analytic Hierarchy Process for decisions in a complex world*, University of Pittsburgh, RWS Publications, Pittsburgh 2001.

sub-criteria. We use a table to represent the matrix as shown below. Note that all the values are 1 across the diagonal: the same determinants are obviously of the same importance.

$$\boldsymbol{C}=\begin{bmatrix} \mathbf{1} & \boldsymbol{c_{12}} & \boldsymbol{c_{13}} & \boldsymbol{c_{14}} & \boldsymbol{c_{15}} & \boldsymbol{c_{16}} & \boldsymbol{c_{17}} & \boldsymbol{c_{18}} \\ \boldsymbol{c_{21}} & \mathbf{1} & \boldsymbol{c_{23}} & \boldsymbol{c_{24}} & \boldsymbol{c_{25}} & \boldsymbol{c_{26}} & \boldsymbol{c_{27}} & \boldsymbol{c_{28}} \\ \boldsymbol{c_{31}} & \boldsymbol{c_{32}} & \mathbf{1} & \boldsymbol{c_{34}} & \boldsymbol{c_{35}} & \boldsymbol{c_{36}} & \boldsymbol{c_{37}} & \boldsymbol{c_{38}} \\ \boldsymbol{c_{41}} & \boldsymbol{c_{42}} & \boldsymbol{c_{43}} & \mathbf{1} & \boldsymbol{c_{45}} & \boldsymbol{c_{46}} & \boldsymbol{c_{47}} & \boldsymbol{c_{48}} \\ \boldsymbol{c_{51}} & \boldsymbol{c_{52}} & \boldsymbol{c_{53}} & \boldsymbol{c_{54}} & \mathbf{1} & \boldsymbol{c_{56}} & \boldsymbol{c_{57}} & \boldsymbol{c_{58}} \\ \boldsymbol{c_{61}} & \boldsymbol{c_{62}} & \boldsymbol{c_{63}} & \boldsymbol{c_{64}} & \boldsymbol{c_{65}} & \mathbf{1} & \boldsymbol{c_{67}} & \boldsymbol{c_{68}} \\ \boldsymbol{c_{71}} & \boldsymbol{c_{72}} & \boldsymbol{c_{73}} & \boldsymbol{c_{74}} & \boldsymbol{c_{75}} & \boldsymbol{c_{76}} & \mathbf{1} & \boldsymbol{c_{78}} \\ \boldsymbol{c_{81}} & \boldsymbol{c_{82}} & \boldsymbol{c_{83}} & \boldsymbol{c_{84}} & \boldsymbol{c_{85}} & \boldsymbol{c_{86}} & \boldsymbol{c_{87}} & \mathbf{1} \end{bmatrix}$$

The following section shows the calculation process of the weights of criteria in Iceland, while the other data are presented in the Appendix.

name of sub-criterion	number
Cost	1
Weight	2
bottom surface area	3
height	4
continuous power rating	5
instantaneous power rating	6
efficiency	7
capacity	8

	1	2	3	4	5	6	7	8
1	1	1.6	1.3	1.9	0.625	0.769231	0.588235	0.5
2	0.625	1	0.769231	0.833333	0.526316	0.555556	0.5	0.333333
3	0.769231	1.3	1	1.5	0.555556	0.625	0.5	0.333333
4	0.526316	1.2	0.666667	1	0.333333	0.5	0.25	0.2
5	1.6	1.9	1.8	3	1	1.3	0.769231	0.588235
6	1.3	1.8	1.6	2	0.769231	1	0.666667	0.526316
7	1.7	2	2	4	1.3	1.5	1	0.769231
8	2	3	3	5	1.7	1.9	1.3	1

The eigenvector **v** matching the maximum eigenvalue λ_{max} of the pairwise comparison matrix **A** is the final expression of the preferences between the investigated elements. The problem of determining the eigenvector leads to the solution of matrix.

A's characteristic equation is shown by:

$$\mathbf{f(\lambda)=|A-\lambda I|}=\begin{bmatrix} \mathbf{1-\lambda} & \boldsymbol{c_{12}} & \cdots & \boldsymbol{c_{17}} & \boldsymbol{c_{18}} \\ \boldsymbol{c_{21}} & \mathbf{1-\lambda} & & & \boldsymbol{c_{28}} \\ \vdots & & \ddots & & \vdots \\ \boldsymbol{c_{71}} & & & \mathbf{1-\lambda} & \boldsymbol{c_{78}} \\ \boldsymbol{c_{81}} & \boldsymbol{c_{82}} & \cdots & \boldsymbol{c_{87}} & \mathbf{1-\lambda} \end{bmatrix}$$

It could be easily indicated that the matrix we use is a positive reciprocal matrix instead of a consistent matrix. A consistent matrix would have the following rules: $\mathbf{C}_{ij} * \mathbf{C}_{jk} = \mathbf{C}_{ik}$. The values of variation of our matrix clearly do not fit the requirements. Our matrix could only be defined as a positive reciprocal matrix.

What's more, the matrix, final resulting vector and the maximum eigenvalue λ_{max} would have the following relation:

$$\mathbf{Av} = \lambda_{max}\mathbf{v}$$

where A is the matrix, v is the resulting vector

We'll later use this equation to evaluation the inconsistency of this method.

By reducing the matrix using Saaty's method, we could get the result of the weight vector **v**. First, the matrix needs processing. Each element in the matrix is divided by the sum of the column containing this element. Then, the weights of each variable could be calculated by the average of each row in reduced matrix. Shown by the equation below. The sum of each column is shown by $\mathbf{a}_j$

$$\begin{bmatrix} \frac{c_{11}}{a_1} & \frac{c_{12}}{a_2} & \frac{c_{13}}{a_3} & \frac{c_{14}}{a_4} & \frac{c_{15}}{a_5} & \frac{c_{16}}{a_6} & \frac{c_{17}}{a_7} & \frac{c_{18}}{a_8} \\ \frac{c_{21}}{a_1} & \frac{c_{22}}{a_2} & \frac{c_{23}}{a_3} & \frac{c_{24}}{a_4} & \frac{c_{25}}{a_5} & \frac{c_{26}}{a_6} & \frac{c_{27}}{a_7} & \frac{c_{28}}{a_8} \\ \frac{c_{31}}{a_1} & \frac{c_{32}}{a_2} & \frac{c_{33}}{a_3} & \frac{c_{34}}{a_4} & \frac{c_{35}}{a_5} & \frac{c_{36}}{a_6} & \frac{c_{37}}{a_7} & \frac{c_{38}}{a_8} \\ \frac{c_{41}}{a_1} & \frac{c_{42}}{a_2} & \frac{c_{43}}{a_3} & \frac{c_{44}}{a_4} & \frac{c_{45}}{a_5} & \frac{c_{46}}{a_6} & \frac{c_{47}}{a_7} & \frac{c_{48}}{a_8} \\ \frac{c_{51}}{a_1} & \frac{c_{52}}{a_2} & \frac{c_{53}}{a_3} & \frac{c_{54}}{a_4} & \frac{c_{55}}{a_5} & \frac{c_{56}}{a_6} & \frac{c_{57}}{a_7} & \frac{c_{58}}{a_8} \\ \frac{c_{61}}{a_1} & \frac{c_{62}}{a_2} & \frac{c_{63}}{a_3} & \frac{c_{64}}{a_4} & \frac{c_{65}}{a_5} & \frac{c_{66}}{a_6} & \frac{c_{67}}{a_7} & \frac{c_{68}}{a_8} \\ \frac{c_{71}}{a_1} & \frac{c_{72}}{a_2} & \frac{c_{73}}{a_3} & \frac{c_{74}}{a_4} & \frac{c_{75}}{a_5} & \frac{c_{76}}{a_6} & \frac{c_{77}}{a_7} & \frac{c_{78}}{a_8} \\ \frac{c_{81}}{a_1} & \frac{c_{82}}{a_2} & \frac{c_{83}}{a_3} & \frac{c_{84}}{a_4} & \frac{c_{85}}{a_5} & \frac{c_{86}}{a_6} & \frac{c_{87}}{a_7} & \frac{c_{88}}{a_8} \end{bmatrix}$$

The calculation of $\mathbf{a}_j$is presented below.

$$a_{\mathrm{j}} = \sum_{\mathrm{i}=1}^{8} c_{\mathrm{ij}}$$

$$(\mathrm{j} = 1,2,3,4,5,6,7,8)$$

Furthermore, the formula of weight vector v is shown below

$$\mathbf{v} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}$$

In the calculation of vector v, w is the weight of each variable, which is shown as:

$$w_{\mathrm{i}} = \frac{\sum_{\mathrm{j}=1}^{8} c_{\mathrm{ij}}}{8}$$

By applying the case of Iceland to the models constructed, we obtain the reduced matrix shown by the table:

0.105036	0.115942	0.10712	0.098787	0.091784	0.094387	0.105529	0.117635

0.065647	0.072464	0.063385	0.043328	0.077292	0.068168	0.0897	0.078423
0.080797	0.094203	0.0824	0.07799	0.081586	0.076689	0.0897	0.078423
0.055282	0.086957	0.054933	0.051993	0.048952	0.061351	0.04485	0.047054
0.168058	0.137681	0.14832	0.155979	0.146855	0.159513	0.138	0.138394
0.136547	0.130435	0.13184	0.103986	0.112965	0.122703	0.1196	0.123826
0.178561	0.144928	0.1648	0.207972	0.190912	0.184054	0.1794	0.180976
0.210072	0.217391	0.247201	0.259965	0.249654	0.233135	0.23322	0.235269

As a result, the weights for each variable could be calculated:

Variable	Weights
Cost	0.1045275
Weight	0.0698009
bottom surface area	0.0827235
height	0.0564215
continuous power rating	0.1491001
instantaneous power rating	0.1227378
efficiency	0.1789504
capacity	0.2357384

The weights we obtained are presented in the pie-chart:
(Specifically, weights on variable for choosing the battery type in Iceland)

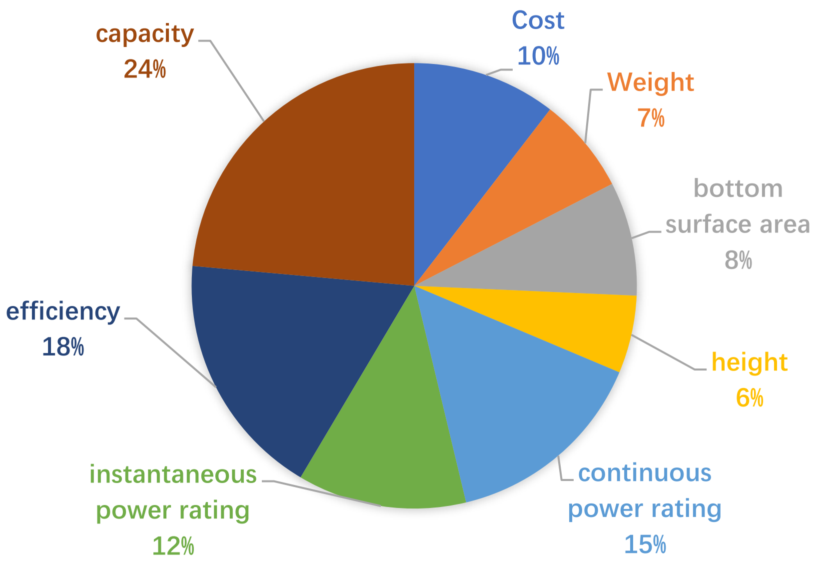


Figure 3. Pie chart of weights for criteria

5.3 Evaluation for consistency

As is discussed above, since the matrix does not meet the requirement of $\mathbf{C}_{ij} * \mathbf{C}_{jk} = \mathbf{C}_{ik}$, the matrix does not belong to a consistent matrix. It is just a positive reciprocal matrix. As a result, there would be inconsistency in the calculation of the weight vector, also the eigenvector $\mathbf{v}$. By calculating the consistent index, CI, the accuracy could be determined. In general, if CI ≤ 0.01, we could indicate that the result is reliable.

The first step of calculating for inconsistency is to conduct the matrix multiplication. **A** represent the original matrix and **v** is the weight vector. The original matrix has 8 columns and 8 rows, and the vector has 8 rows. Consequently, the result would also be a vector with 8 rows, according to the multiplication of the matrix. Shown by the equation below.

$$\mathbf{Av} = \begin{bmatrix} 1 & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} & c_{18} \\ c_{21} & 1 & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} & c_{28} \\ c_{31} & c_{32} & 1 & c_{34} & c_{35} & c_{36} & c_{37} & c_{38} \\ c_{41} & c_{42} & c_{43} & 1 & c_{45} & c_{46} & c_{47} & c_{48} \\ c_{51} & c_{52} & c_{53} & c_{54} & 1 & c_{56} & c_{57} & c_{58} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & 1 & c_{67} & c_{68} \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & 1 & c_{78} \\ c_{81} & c_{82} & c_{83} & c_{84} & c_{85} & c_{86} & c_{87} & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

After conducting the multiplication of the matrix. Using the equation above to make the calculation of the inconsistency,

$$\mathbf{Av} = \lambda_{max}\mathbf{v}$$

We should divide every element in the **Av** vector by each weight in the according rows to make a new vector **x**.

$$\mathrm{x} = \begin{bmatrix} \frac{x_1}{w_1} \\ \frac{x_2}{w_2} \\ \frac{x_3}{w_3} \\ \frac{x_4}{w_4} \\ \frac{x_5}{w_5} \\ \frac{x_6}{w_6} \\ \frac{x_7}{w_7} \\ \frac{x_8}{w_8} \end{bmatrix}$$

Then the maximum eigenvalue λ_{max} could be calculated by the average of each element in the vector.

$$\boldsymbol{\lambda} = \frac{\sum_{i=1}^{8} \frac{x_i}{w_i}}{8}$$

The Consistency Index (CI) could be calculated in the equation:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

In our Iceland case, the processing data is shown below.

x	**x_i/w_i**
0.841686	8.052287
0.560498	8.029957
0.666102	8.052147

0.4533	8.034175
1.203015	8.068511
0.987531	8.045863
1.446656	8.084116
1.903781	8.075823

$$\boldsymbol{\lambda = \frac{\sum_{i=1}^{8} \frac{x_i}{w_i}}{8} = 8.05536}$$

$$\boldsymbol{CI = \frac{\lambda_{max} - n}{n - 1} = \frac{8.05536 - 8}{8 - 1} = 0.007909}$$

It could be indicated that the result of CI from Iceland is clearly less than 0.01, so that our calculation in AHP would be valid. It indicates that our method is feasible.

What’s more, in AHP’s method, besides CI, people also invented Consistency Ration (CR) in order to evaluate whether the pairwise comparisons are affecting the result. The CI values for random pairwise comparisons (r) should vary considerably from the experts’ estimates. The expression of this difference is the Consistency Ratio (CR).[7]

$$\boldsymbol{CR = \frac{CI}{random\ average\ CI} = \frac{\lambda_{max} - n}{r(n - 1)} \times 100\%}$$

The random average CI has the criteria below:

$$\boldsymbol{CR = \frac{\lambda_{max} - n}{r(n - 1)} \times 100\% = \frac{8.05536 - 8}{1.40(8 - 1)} \times 100\% = 0.5649\%}$$

Therefore, our estimates are deemed acceptable since CR is less than 10%.

5.4 Evaluation for AHP

5.4.1 Strength

- Our model simplifies the decision-making process by providing a clear ranking system and a direct rating system to measure the weights and priority of battery properties, which is helpful for ordinary consumers without professional modeling knowledge.
- Our model contains all eight quality measures of batteries, which is comprehensive and realistic.
- Our selected case from 9 locations over the world are representative and comprehensive of major climate types, and thus provide more general and applicable choice to satisfy personalized demand.
- The results yielded by our model coincides with the suggestions offered by professional engineers in related fields from online.

5.4.2 Weakness

- The model inherited the subjectivity of AHP analysis, which adds to the

---

[7] Random average consistency indexes are calculated based on a large number (e.g., 1000) of randomly generated matrices. The values of *r* given in the literature may therefore vary to an insignificant degree.

Cabala, P. (2010, January). Using the Analytic Hierarchy Process in Evaluating Decision Alternatives. *Operations Research and Decisions 1(No. 1):1–23.* Published.

imprecisions and limitations of our results obtained.

- Although we took several geographic factors under considerations, the average values of electricity consumptions and maximum loads are calculated based on statistics in America merely. This may violate the universality of our model, for an incomprehensive set of database.
- We have a relatively small sample capacity. The five batteries may not provide the best solution for each family. Another drawback it brings is that the small sample size might not reveal potential loopholes of our model.

6. The EWM model

6.1 Basic info

The Entropy Weight Method (EWM) utilizes the variation of the data to determine the importance of the various factors. For a given factor, the larger the variation, the higher the entropy. A higher entropy will mean that the data is more uncertain, and it will account for a smaller weight.

6.2 EWM model

6.2.1 Matrix

For a certain battery, there are various factors that accounts to the quality of the battery. A matrix is created from original data. The matrix has 5 rows and 8 columns for the 5 batteries and the 8 criteria.

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} \\ x_{51} & x_{42} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} \end{bmatrix}$$

6.2.2 Calculation

For the instantaneous power rating of the battery, some of the data are not available, so this criterion is removed.

Due to the different meaning of positive factors (the bigger the better) and negative factors (the smaller the better), two different methods of calculations are used to standardize the data.

For positive factors,

$$z_{\mathrm{ij}} = \frac{x_{\mathrm{ij}} - \min(x_{\mathrm{j}})}{\max(x_{\mathrm{j}}) - \min(x_{\mathrm{j}})}$$

For negative factors,

$$z_{\mathrm{ij}} = \frac{\max(x_{\mathrm{j}}) - x_{\mathrm{ij}}}{\max(x_{\mathrm{j}}) - \min(x_{\mathrm{j}})}$$

Then, calculate the weight of x_{ij} in the factor j

$$\mathcal{P}_{\mathrm{ij}} = \frac{z_{\mathrm{ij}}}{\sum_{\mathrm{i}=1}^{7} z_{\mathrm{ij}}}$$

The entropy of the data can then be calculated. The constant k is equal to the reciprocal of the natural logarithm of the amount of data for a factor.

$$e_{\mathrm{j}} = -\mathrm{k} \sum_{\mathrm{i}=1}^{7} \mathcal{P}_{\mathrm{ij}} \ln(\mathcal{P}_{\mathrm{ij}})$$

$$\mathrm{k} = \frac{1}{\ln 5}$$

As a result, the information utility value and the overall weight of the factors can be determined from the entropy value.

$$d_{\mathrm{j}} = 1 - e_{\mathrm{j}}$$

$$w_{\mathrm{j}} = \frac{d_{\mathrm{j}}}{\sum_{\mathrm{j}=1}^{7} d_{\mathrm{j}}}$$

The overall weight vector of the factors is:

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ 0 \\ w_6 \\ w_7 \end{bmatrix}$$

The resultant weights of the factors are:

Variables	Weights
Cost	0.168154078
Weight	0.115377985
BSA	0.166608753
Height	0.177537271
CPR	0.122137235
IPR	0
Efficiency	0.107709148
Capacity	0.14247553

These weights can be shown by the graph below.

6.3 Combining the AHP method and the EWM method

Since both the AHP method and the EWM method is used in determining the

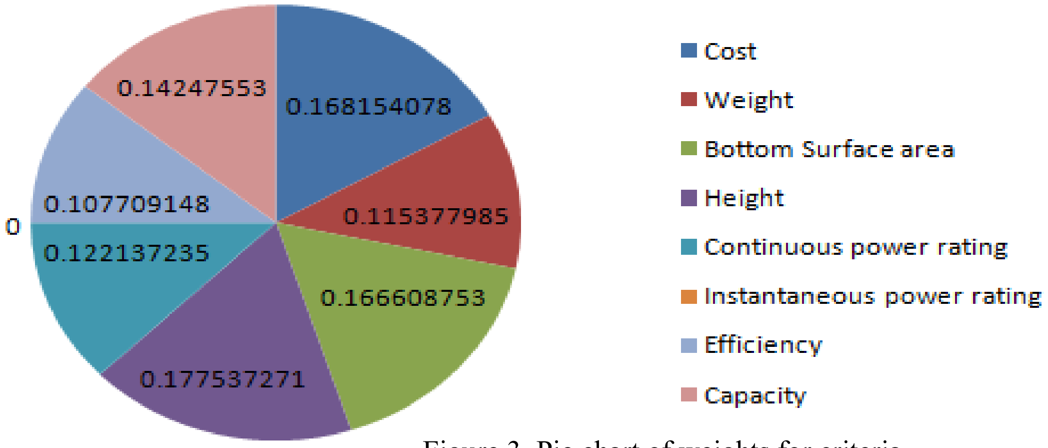


Figure 3. Pie chart of weights for criteria

weight of the factors, a method needs to be determined to combine the two methods. To minimize the difference between the weights calculated by the AHP method and the EWM method, the weight for w_{AHP} and w_{EWM} is determined so that

$$\sum_{j=1}^{8}(w_1 w_{\mathrm{AHP}} - w_2 w_{\mathrm{EWM}})$$

is minimal. w_1 and w_2 represents the weight of the AHP model and the weight of the EWM method respectively.

The balanced weights of the factors are

Variables	Weights
Cost	0.218153266
Weight	0.113212496
BSA	0.1119955
Height	0.108913902
CPR	0.151746672
IPR	0.027740961
Efficiency	0.136110562
Capacity	0.132126641

and the overall ranks of the batteries are

Battery	Rank
Deka Solar 8GCC2 6V 198	2
Trojan L-16 -SPRE 6V 415	4
Discover AES 7.4 kWh	1
Electriq PowerPod 2	5
Tesla Powerwall+	3

6.4 Evaluation

The EWM model does reduce the objectivity of the AHP model, but it does not consider the relative importance of the factors. For example, the method gives a 0.17 weight to height while only giving 0 to the instantaneous power rating of the battery. As discussed before, the height of the battery is relatively not important while the instantaneous power rating is an important factor. Therefore, we decided not to include the EWM model in further calculation.

7. Application of Concrete Battery

(For question 3)

7.1 General description

Concrete—a mixture of specific proportion of water, cement and aggregate (like gravel and sand)—has always been an ideal and ubiquitous construction material with high durability and hardness since it was invented by Ancient Roman. A recent study conducted by researchers at Chalmers University of Technology in Sweden has provided an additional application for concrete to address the challenge of sustainable future; that is, the improved version of cement-based battery.

The research team used the layer structure of the battery, consisting of an anode layer, a cathode layer, and between them, a separator layer[8]. The separator layer, or the electrolyte of the battery, is a mixture of cement and carbon fiber (CF) electroplated with metal[9]. After experimenting with numerous metals, iron and nickel yielded the greatest energy density when coated outside anodes and cathodes[10].

7.2 Evaluation

Assumption

Given that the house is 1600 square foot in size, we assume it has one floor with length and width of 40.0ft*40.0ft (or 12.2m*12.2m). Height is fixed as 3.00 meters or 9.83 feet. Thickness of concrete layer would be at least 6.00 inches to meet government requirement[11]. No interior walls, doors, or windows are considered to simplify the model. Solar panels are installed to convert solar energy to electricity. We would like to incorporate the cement-based batteries into exterior walls and the ceiling.

7.2.1 Cost

To estimate the cost of concrete-based battery for the given household, we first calculate the volume of cement to fill in walls and ceilings. In the formula below, Lrepresents length and width; h stands for wall height, and ∂ stands for thickness. V is the minimum total volume of concrete.

$$V = 4L \times h \times \partial + L \times W \times \partial$$
$$\therefore V = 44.286 \approx 44.29\, m^3$$

The costs of metal mesh and metal powder in the electrodes ought to be added, yet the size and thickness of which are unknown. Meanwhile, the processing charge of electroplating and other technical steps should also be considered. Furthermore, additional protection layer against air exposure and human contact may be necessary for the long run. Overall, this imposes greater financial burden on construction compared to the conventional wooden structure. Consumers may prefer the cheaper and convenient way instead of experimenting with this novel technology, unless crucial scientific breakthroughs in the future is able to lower the installation costs. Though, compared to purchasing

7.2.2 Efficiency

From the information and data provided by the research team (energy density equals to 0.8Wh/L[12]) we may estimate the battery's capacity if installed in this 1600 square feet house. Energy density is the amount of energy a battery may contain in relation to its volume. Assuming the battery's volume equals to that of the concrete as we calculated above, we may obtain the resultant capacity by the formula shown below. D and V are respectively energy density and concrete volume.

$$E = D \times V = 0.8WhL^{-1} \times 44.29L = 35.36Wh$$

The energy capacity is far less than that of other batteries given as sample in the

---

[8] Zhang, Emma Q., and Luping Tang. 2021. "Rechargeable Concrete Battery" *Buildings* 11, no. 3: 103. https://doi.org/10.3390/buildings11030103

[9] Ibid., 2

[10] Ibid., 13

[11] Chapter 5- Foundation Requirements, "HUD Foundation Requirements Manufactured Home." https://www.hud.gov/sites/documents/49030GC5GUID.PDF

[12] ibid., 13

question and may not be capable of sustaining continuous electricity use. Yet, if accompanied with the installation of other batteries, the cement-based battery may provide extra but little energy while fully using the space provided. Furthermore, for larger architectures, there would be more energy stored in the cement-based battery, providing alternative storage of electricity while being space-efficient.

7.3 Missing Information

7.3.1 Durability and repairment

In order to become applicable in real life, the durability of the cement-based batteries should be considered. As the battery is preserved inside the walls and covered with thick thermal shroud, it would be impossible to replace the dysfunctional components unless the walls are being teared down, which is cost-demanding and dangerous for families. Such problem may never be solved in densely populated cities, whereas the cost and term of repairing may exceed residents' purchasing power. Possibilities of metal electrodes undergoing chemical reaction with oxygen to produce metal oxides hamper the battery's capacity over the long run. Thus, the cement-based batteries should be capable of enduring extreme weathers and be sustainable over very long time to be commercially available. Yet, experiments of this aspect were not conducted.

7.3.2 Effects on health

To become commercialized, the batteries' potential effects and harm on human health should be thoroughly investigate by researchers. Ordinary off-grid batteries are required to pass certain quality examinations before launching to avoid damage on health resulted from ionizing radiation. Furthermore, researchers should provide suggestions on protection against potential dangers, including human contact with batteries or leakage of electricity.

7.3.3 Performance under extreme weathers

Presumably, the experiment led by the Swedish research team was carried under room temperature and pressure, though specific description was not offered. Yet, for the marketing and commercialization of the cement-based batteries worldwide, their performances (including storage capacity, continuous power rating, durability, etc.) under various climates, temperature, and atmospheric temperature ought to be investigated. For example, how would the batteries' function be affected by air humidity or direct contact with water in tropical or arid regions? Also, would the harsh freezing temperature near the Arctic circle influence the capacity and discharge of batteries? Such information is necessary for the decision-making process of consumers with drastically different backgrounds and demands.

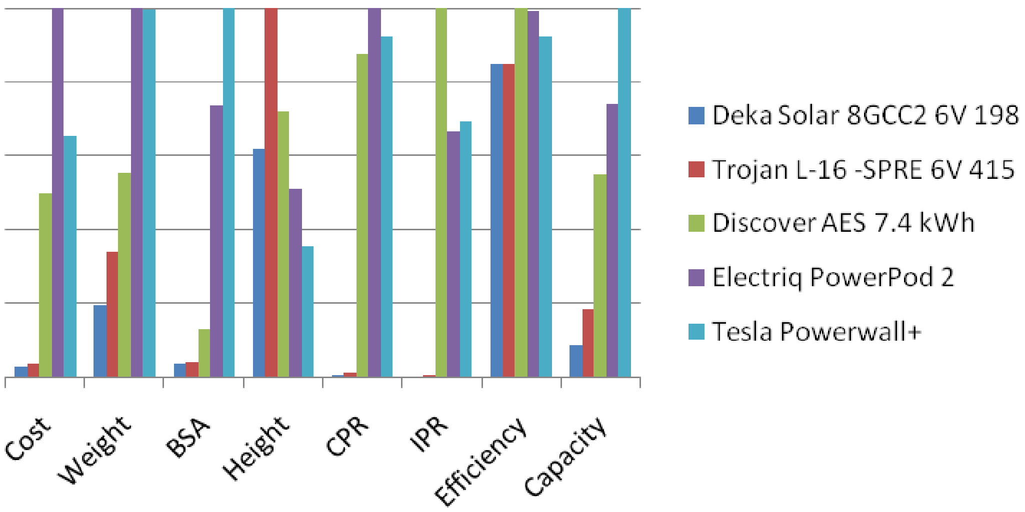
8. Conclusion

In short, we believe that the Discover AES 7.4kWh battery is the best battery due to its excellent instantaneous power ratings and round trip efficiency. Its continuous power rating and capacity is also good while having medium cost. This would be of use both in warm countries, where the instantaneous power rating is necessary to power air conditioners; and in colder countries, where capacity and efficiency are important in making the most out of the minimal sunshine. The choice of this battery, therefore, would be the best choice.

Another alternative to this choice is the Tesla Powerwall + battery. It also has good qualities and excellent capacity. However, due to its higher cost and bulkiness, it isn't as good as the Discover battery. The rest of the batteries were not a good choice, either because of poor quality or high cost.

A graph of the comparative qualities of the batteries is shown below.

Figure 4. Relative quantities for factors



By far, we have decided the optimal choice: Discover AES 7.4kWh. The suggestions provided by researchers and scientists show high correspondence with the solution that the algorithm had provided: Discover AES 7.4kWh is the most recommended battery among the five.

To begin with, the first two lead acid batteries are obsolete products with several disadvantages. Modern batteries like LFP (Lithium Iron Phosphate) and NMC (Lithium Nickel Manganese Cobalt Oxide) outperform them in every aspect, from capacity to discharging rate. The lead acid batteries also cause great harm to the environment, for the leakage of heavy metal elements like lead is irreversible. Nevertheless, the most lethal drawback lies in its durability. A lead acid battery is not capable of functioning after 1000 cycles, which is approximately a five-year period. Considering the fact that solar panels need to be renovated once every 25 years, the five renewals of the batteries would lead to huge costs. Thus, the first two lead acid batteries are not qualified.

The Tesla Powerwall, on the other hand, is one of the mainstreams of energy storage industry. As the leading innovation of Tesla, it is renowned for substantial energy density, with excellent capacity and discharging rate. Generally, greater costs accompany better performance. However, the crucial defect of NMC batteries lies in the security part. Usually, energy storage system for civil use does not equip a temperature monitoring device. Once a thermal runaway takes place, the temperature of the battery can easily exceed 200 degrees, causing massive explosions.

In December 2020, LG Chem initiated a large-scale recall of Resu 10H energy storage battery for home use. It was contended that there were cases of overheat that may result in generating harmful smoke and fire. Previously, 5 flammations had taken place in America, and batteries produced in 2017 were recalled, generating extra cost of 358 million dollars. The active Nickel contributes to both a high energy transferring rate and a potential risk.

The battery type that satisfies all needs is targeted to be LFP. They are potent competitors in both power ratings and capacity. It is also said that NMC batteries are usually 20% more costly than the LFP ones. The greatest advantage of LFP batteries is also the life span. While an NMC battery can perform approximately 3000 charging cycles, LFP batteries take more than 6000 cycles, with an expected working age of 20 years. The aggregation of the superiority makes LFP the undoubted optimal battery type. It is suggested that the upgrade in capacity is not comparable to the doubled price of ElectriqPowerPod. Therefore, the Discover AES 7.4 kWh is the most appropriate battery type, proving the accuracy of our algorithm.

9. Appendix

9.1 Reference

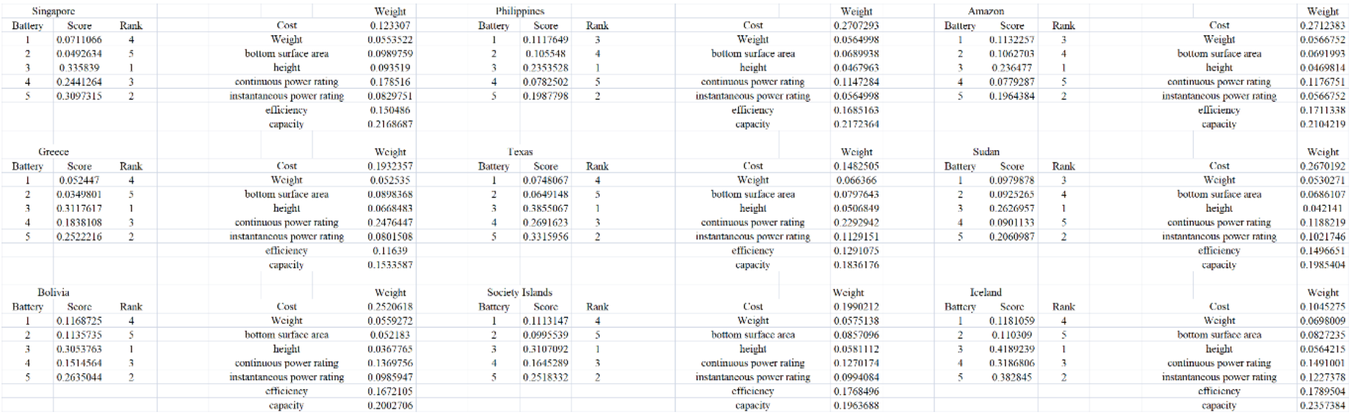
1. [1] CNBC. Ellyat, Holly. 2021 Nov 9th. *COP26 climate summit switches to science and innovation.* https://www.cnbc.com/2021/11/09/cop26-updates-as-climate-summit-continues-in-glasgow.html?&qsearchterm=COP26
2. [2] US Energy Information Administration. 2021 Oct 7th . *How much electricity does an American home use?* https://www.eia.gov/consumption/residential/data/2015/hc/php/hc9.4.php
3. [3] US Energy Information Administration. *Residential Energy Consumption Survey (RECS)* https://www.eia.gov/tools/faqs/faq.php?id=97&t=3
4. [4] Electrical Load Calculation, Online Interactive Residential Electrical Load Calculator https://ask-the-electrician.com/residential-electrical-load-calculation.html#beginAdv
5. [5] Krishnamurti, T. (2015, October 12). *Indian monsoon*. Encyclopedia Britannica. https://www.britannica.com/science/Indian-monsoon
6. [6] SAATY T., *Decision making for Leaders: The Analytic Hierarchy Process for decisions in a complex world*, University of Pittsburgh, RWS Publications, Pittsburgh 2001.
7. [7] Cabala, P. (2010, January). Using the Analytic Hierarchy Process in Evaluating Decision Alternatives. *Operations Research and Decisions 1(No. 1):1–23.* Published.
8. [8] Zhang, Emma Q., and Luping Tang. 2021. “Rechargeable Concrete Battery” *Buildings* 11, no. 3: 103. https://doi.org/10.3390/buildings1103010
9. [9] Chapter 5- Foundation Requirements, “HUD Foundation Requirements Manufactured Home.”

https://www.hud.gov/sites/documents/49030GC5GUID.PDF

9.2 Program and Data

```python
class matrix():
#Input the row and column form of a matrix, and the count of rows and columns
    def __init__(self,value):
        self.rows=value
        self.row_count=len(value)
        rev=[]
        for i in range(len(value[0])):
            cur=[]
            for j in range(len(value)):
                cur.append(value[j][i])
            rev.append(cur)
        self.columns=rev
        self.column_count=len(self.columns)
#Get the average value for each row
    def get_average(self):
        ret=[]
        for i in range(len(self.rows)):
            ret.append(sum(self.rows[i])/len(self.rows[i]))
        return ret
#Reduce the value for each column by dividing by the sum of the column
    def get_proportion(self):
        ret=[]
        for i in range(len(self.rows)):
            cur=[]
            for j in range(len(self.columns)):
                cur.append(self.rows[i][j]/sum(self.columns[j]))
            ret.append(cur)
        ret=matrix(ret)
        return ret
#Reduce the value for each column by dividing by the maximum value in the column
    def get_proportion_maximum(self):
        ret=[]
        for i in range(self.row_count):
            cur=[]
            for j in range(self.column_count):
                cur.append(self.rows[i][j]/max(self.columns[j]))
            ret.append(cur)
        return matrix(ret)
#Calculate the sum of each row
    def get_sum(self):
        ret=[]
        for i in range(self.column_count):
            ret.append(sum(self.rows[i]))
        return ret
    def __setitem__(self,pos,value):
        self.rows[pos[1]][pos[0]]=value
    def __str__(self):
        return str(self.rows)

#Multiply the values of the first matrix with the corresponding value in the second matrix
def multiplymatrix(m1,m2):
    output=[]
    for i in range(m1.row_count):
        cur=[]
        for j in range(m1.column_count):
            cur.append(m1.rows[i][j]*m2.rows[i][j])
        output.append(cur)
    return matrix(output)
#Multiply the values of the matrix with the corresponding value in the list
def multiplymatrixlist(m,l):
    output=[]
    for i in range(m.row_count):
        cur=[]
        for j in range(m.column_count):
            cur.append(m.rows[i][j]*l[j])
        output.append(cur)
    return matrix(output)
```



Singapore				Weight
Battery	Score	Rank	Cost	0.123307
1	0.0711066	4	Weight	0.0553522
2	0.0492634	5	bottom surface area	0.0989759
3	0.335839	1	height	0.093519
4	0.2441264	3	continuous power rating	0.178516
5	0.3097315	2	instantaneous power rating	0.0829751
			efficiency	0.150486
			capacity	0.2168687

Philippines				Weight
Battery	Score	Rank	Cost	0.2707293
1	0.1117649	3	Weight	0.0564998
2	0.105548	4	bottom surface area	0.0689938
3	0.2353528	1	height	0.0467963
4	0.0782502	5	continuous power rating	0.1147284
5	0.1987798	2	instantaneous power rating	0.0564998
			efficiency	0.1685163
			capacity	0.2172364

Amazon				Weight
Battery	Score	Rank	Cost	0.2712383
1	0.1132257	3	Weight	0.0566752
2	0.1062703	4	bottom surface area	0.0691993
3	0.236477	1	height	0.0469814
4	0.0779287	5	continuous power rating	0.1176751
5	0.1964384	2	instantaneous power rating	0.0566752
			efficiency	0.1711338
			capacity	0.2104219

Greece				Weight
Battery	Score	Rank	Cost	0.1932357
1	0.052447	4	Weight	0.052535
2	0.0349801	5	bottom surface area	0.0898368
3	0.3117617	1	height	0.0668483
4	0.1838108	3	continuous power rating	0.2476447
5	0.2522216	2	instantaneous power rating	0.0801508
			efficiency	0.11639
			capacity	0.1533587

Texas				Weight
Battery	Score	Rank	Cost	0.1482505
1	0.0748067	4	Weight	0.066366
2	0.0649148	5	bottom surface area	0.0797643
3	0.3855067	1	height	0.0506849
4	0.2691623	3	continuous power rating	0.2292942
5	0.3315956	2	instantaneous power rating	0.1129151
			efficiency	0.1291075
			capacity	0.1836176

Sudan				Weight
Battery	Score	Rank	Cost	0.2670192
1	0.0979878	3	Weight	0.0530271
2	0.0925265	4	bottom surface area	0.0686107
3	0.2626957	1	height	0.042141
4	0.0901133	5	continuous power rating	0.1188219
5	0.2060987	2	instantaneous power rating	0.1021746
			efficiency	0.1496651
			capacity	0.1985404

Bolivia				Weight
Battery	Score	Rank	Cost	0.2520618
1	0.1168725	4	Weight	0.0559272
2	0.1135735	5	bottom surface area	0.052183
3	0.3053763	1	height	0.0367765
4	0.1514564	3	continuous power rating	0.1369756
5	0.2635044	2	instantaneous power rating	0.0985947
			efficiency	0.1672105
			capacity	0.2002706

Society Islands				Weight
Battery	Score	Rank	Cost	0.1990212
1	0.1113147	4	Weight	0.0575138
2	0.0995539	5	bottom surface area	0.0857096
3	0.3107092	1	height	0.0581112
4	0.1645289	3	continuous power rating	0.1270174
5	0.2518332	2	instantaneous power rating	0.0994084
			efficiency	0.1768496
			capacity	0.1963688

Iceland				Weight
Battery	Score	Rank	Cost	0.1045275
1	0.1181059	4	Weight	0.0698009
2	0.110309	5	bottom surface area	0.0827235
3	0.4189239	1	height	0.0564215
4	0.3186806	3	continuous power rating	0.1491001
5	0.382845	2	instantaneous power rating	0.1227378
			efficiency	0.1789504
			capacity	0.2357384